Educational Goals for Quantitative Literacy

> Quantitative literacy describes a habit of mind rather than a set of topics or a list of skills. ... [QL] is not about how much mathematics a person knows, but about how well it can be used.
>
> — Deborah Hughes Hallett

> An important part of [QL] is using, doing, and recognizing mathematics in a variety of situations. —Jan De Lange

> The QL skills of interpreting and discussing data and then presenting information in a coherent manner are absolutely essential if our young people are going to be successful in this new world of technology.
>
> —Charlotte Frank

As literacy implies an ability to use words to comprehend and express ideas, numeracy implies a similar capacity to employ numbers and grasp the concepts they represent. Both involve interpretation, understanding, and the power of language. Both mature as students move from one tier of education to the next. Both are essential for democratic life in the information age. And both must be recognized as equal but parallel goals of undergraduate education.

As with reading and writing, the third R can be explored on many different levels. At the lowest level, numeracy is exemplified by common experiences such as determining how much paint or carpet to purchase or deciding which cell phone plan is best. Yet even these contexts are usually much more complex in practice than one might suspect from their simplistic caricatures in school texts or standardized tests. There is a very large difference in sophistication, for example, between calculating the carpet needed for a simple rectangular room (a common test question) and calculating the carpet required for adjoining L-shaped rooms when the carpet has both horizontal and vertical repeat patterns. Both require just simple arithmetic, yet the latter—the more realistic case—involves careful reasoning with scale drawings in order to determine just what calculations are appropriate.

The difference in cognitive complexity between carpeting a rectangular room and measuring carpet for realistic floor plans corresponds in curricular terms to the difference between primary and secondary education. Although one rarely sees problems such as realistic carpet measurement among the stated goals of K–12 mathematics education—factoring polynomials and solving quadratic equations are much more common expectations—the problem-solving demands are approximately the same in both. Laying carpet and solving quadratics are both algorithmic: they require only careful application of a series of steps that can themselves be practiced separately and which, when combined in the proper order, will yield a solution to the problem at hand.

Important as these algorithmic abilities may be for daily life, quantitative literacy is not really about them but about challenging college-level settings in which quantitative analysis is intertwined with political, scientific, historical, or artistic contexts. Here QL adds a crucial dimension of rigor and thoughtfulness to many of the issues commonly addressed in undergraduate education.[15]

Like writing, QL has many faces, each suited to different purposes. Unlike biology or mathematics, QL is not a discipline but a literacy, not a set of skills but a habit of mind. As physics and finance depend on mathematical tools, so too does quantitative literacy. But as these disciplines differ from mathematics, so too does quantitative literacy.

Examples of important issues that can benefit from quantitative understanding are not hard to find. The sidebar, "Making Representative Democracy Work," outlines the roots of quantitative thinking in the early years of the United States. Contemporary examples appear regularly in news headlines:

- *Political polling.* How can polls be so accurate? Why do they sometimes fail?
- *Clinical trials*: Why is a randomized double blind study the most reliable?
- *Tax policy:* Can lower tax rates yield greater tax revenue?
- *Vaccination strategy*: Ethics of individual vs. societal risks (e.g., smallpox).
- *Investment strategies:* The logic of diversification vs. the psychology of risk.
- *Improving education*: What data are required to infer causation from correlation?
- *Fighting terrorism*: Balancing lives vs. dollars and other incommensurate comparisons.
- *Cancer screening*: Dealing with false positives when disease incidence is low.
- *Building roads*: Why the "tragedy of the commons" often leads to slower traffic.
- *Judging bias*: Dealing with Simpson's paradox in disaggregated data.
- *Clinical trials*: Ethics of using placebos for seriously ill patients.

Except for a very few items (where a graph can substitute for algebraic analysis), thorough knowledge of these examples does not require advanced mathematics. The mathematics undergirding these examples actually represents a rather complete alternative to traditional secondary school mathematics: linear and exponential functions abound, as do graphs, probability, elementary statistics, and exploratory data analysis. Although some traditional topics are hardly present (e.g., trigonometry and binomial theorem), others such as contingency tables and false positives are common in these examples—yet rarely covered in traditional K–12 curricula. A student whose secondary school mathematics included the tools necessary to deal with these QL issues (but without mastery of trigonometry) may be unprepared for calculus but well prepared for the world.

Many Europeans take a broader view of mathematics, arguing that in education the difference between QL and mathematics should be minimal (e.g., de Lange, Niss). Indeed, what the OECD Program for International Student Assessment (PISA) calls mathematical literacy is very similar to what we are calling quantitative literacy, namely, the "capacity to identify, to understand, and to engage in mathematics and to make well-founded judgments about the role that mathematics plays ...[for] life as a constructive, concerned and reflective citizen."[16] Through this new assessment, the international commu-

Making Representative Democracy Work

Representative democracy originated in a numerical conception of the social order, under the US Constitution. That same document ordained that government should "promote the general welfare and secure the blessings of liberty," a mandate that around 1820 was increasingly answered with a turn toward "authentic facts" and statistics. Statistics soon became compressed into quantitative facts, an efficient and authoritative form of information that everyone assumed would help public-spirited legislators govern more wisely. Schools, both public and private, correspondingly stepped up arithmetic instruction for youth, bringing a greatly simplified subject to all school-attending children, making it possible for them to participate with competency both in the new market economy and in the civic pride that resulted from the early focus on quantitative boasting.

As basic numeracy skills spread, so did the domain of number in civic life. The unsophisticated empiricism of early statistical history yielded to a more complex political terrain where numbers were enlisted in service of political debates and strategizing. At mid-century, the level of quantitative mastery required to keep up with debates based on numbers was still within the reach of anyone schooled to long division and percentage calculations. At a deeper level, the quantitative savvy required to challenge numbers (for bias, for errors in measurement and counting, for incorrect comparison of figures, for selective use of numbers) was not well developed, either in the producers or consumers of numbers. Choices about what to count and what not to count might be made naively, or purposefully and politically, as in the decision not to collect comparable demographic data on blacks and whites in the census.

— Patricia Cline Cohen

nity is saying that to serve a broader range of students, mathematics education must become more like QL.

It is easy to identify several key features of quantitative literacy. *Engagement with the world* is the most important. Whereas mathematics can thrive in an abstract realm free from worldly contexts, QL is anchored in specific contexts, often presented through "thick descriptions" with rich and sometimes confusing detail. (Contrast this with the thin gruel of so-called "application problems" common in mathematics textbooks.) Often the contexts of QL are personal or political, involving questions of values and preferences. What is the fairest way to run an election? Are the risks of mass vaccination worth the potential benefits? For students, contexts create meaning. Yet all too often, mathematics students fail to see the relevance of their studies. "If we try to teach students the right competencies but use contexts that are wrong for the students," writes Dutch mathematician Jan de Lange, "we are creating a problem, not solving it."

A second cornerstone of QL is the ability to apply quantitative ideas in *unfamiliar contexts*. "Employees must be prepared to apply quantitative principles in unforeseeable contexts," writes Linda Rosen, former vice president of the National Alliance of Business. This is very different from most students' experience in mathematics courses where the vast majority of problems are of types they have seen before and practiced often. "An essential component of QL is the ability to adapt a quantitative argument from a familiar to an unfamiliar context," observes University of Arizona mathematician Deborah Hughes Hallett. Dealing with the unpredictable requires "a mind searching for patterns rather than following instructions. A quantitatively literate person needs to know some mathematics, but literacy is not defined by the mathematics known."

Third, quantitative literacy insists on flexible understanding that adapts readily to new circumstances. For mathematicians and mathematics teachers, conceptual understanding is the gold standard of mathematical competence. Quantitative literacy requires even more: a flexible understanding that empowers sound judgment even in the absence of sufficient information or in the face of inconsistent evidence. A civil servant timing meter ramps on freeways, a business owner investing in new equipment, and a patient deciding on a treatment for prostate cancer all must make quantitatively based decisions without the kind of full information that they have come to expect from their experience of mathematics in school. The flexibility demanded by QL is not unlike what the Mathematics Learning Study Committee of the National Research Council calls "adaptive reasoning."[17]

Notwithstanding widespread agreement that QL requires engagement with the world, flexible understanding, and capacity to deal with unforeseeable contexts, there still remains some disquiet about the term itself. The facility we want students to acquire is not just about quantities, nor is it as passive as "literacy" may suggest. Reasoning, argument, and insight are as essential to QL as are numbers; so too are notions of space, chance, and data. Like the international team that is designing PISA, many prefer the term "mathematical literacy" since it does not suggest a restriction to the quantitative aspects of mathematics, but invites a role for mathematics in its broadest sense.

The word "context" also causes confusion. Since context is a catalyst for promoting learning, good teachers always provide context. So in some sense, teaching "in context" is redundant: it just means "teaching well." But contexts can be quite varied and only a few may appeal to a particular student at a particular time. So although learning in context is usually easier than learning without context—at least if the context is relevant to the student—teaching in context is not at all easy. As Russell Edgerton observed, the more context dominates content, the more the instructional emphasis can shift from teaching to learning. All too often, unfortunately, feeble attempts at context produce minds that are "not in gear:"

> A quantitatively literate person must be able to think mathematically in context. This requires a dual duty, marrying the mathematical meaning of symbols and operations to their contextual meaning, and thinking simultaneously about both. It is considerably more difficult than the ability to perform the underlying mathematical operations, stripped of their contextual meaning. Nor is it sufficient simply to clothe the mathematics in a superficial layer of contextual meaning. The mathematics must be engaged with the context and providing power, not an engine idling in neutral. Too many attempts at teaching mathematics in context amount to little more than teaching students to sit in a car with the engine on, but not in gear. The "everyday life" test provides a measure of engagement; everyday life that moves forward must have an engine that is in gear.
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> —Randall Richardson and William McCallum

As students move from school to college and beyond, so does their potential need for quantitative literacy. As students move through education and on into adult life, issues they face in finance, politics, and health increase in subtlety and sophistication, yet often rely only on mathematical techniques of ratios, averages, and straight line graphs that were first learned in the middle grades. It is this increased sophistication built on a foundation of relatively elementary mathematics that is a hallmark of quantitative literacy.

Much the same is true for writing: although the principles of grammar and organization are more or less established during school-level instruction, students' sophistication in writing continues to grow through college and well beyond. Experienced educators know approximately what to expect of student writers at different grade levels. This is not yet true, however, for quantitative literacy because in its present form QL is largely new—both as a requirement of engaged living and as a goal of education.

So one important task required to establish QL as an effective goal for undergraduate education is to define what QL means at different educational levels. Rita Colwell, Director of the National Science Foundation, argues that the nation needs "flexible goals" for QL based on standards that are appropriate for different audiences. "Most importantly," she notes, "we must understand that literacy has levels." For QL to take root in higher education, we need to understand and express how college-level QL differs from high school level QL, and how both differ from the middle school canon of simple percentage and interest problems.

Importance of Quantitative Literacy

In our work-based society, failure to give people the knowledge and skills they need to get and keep good jobs can have disastrous personal consequences. —Anthony Carnevale

Careers and work are not the only economic reasons for taking mathematics. Nonetheless, the primary force behind the nation's emphasis on mathematics is economic. — Arnold Packer

Many individuals are not prepared to function effectively in the workplace today because they lack versatility and flexibility on how they approach and deal with obstacles. —Linda Rosen

The case for improved quantitative literacy is often argued in terms of economic competitiveness, both for individuals and for society. While this case is legitimate and strong, a more general issue underlies our concern in this report: the increasing importance of quantitative data for each person's quality of life and for our collective well-being. We see this not just in terms of jobs and workforce issues, but also for everyday issues of personal welfare, social decision-making, and the functioning of democratic society.

Personal welfare includes issues such as health, safety, taxes, budgets, credit, and financial planning. Health and safety are obviously important, as we have seen in the reduced effectiveness of antibiotics, the spread of viruses to new regions of the world, and recently, the threat of bioterrorism. So too are issues of family finance, especially as consumer choices expand for household utilities, health insurance, and retirement plans. Decisions about health and finance virtually always involve quantitative reasoning. They require average citizens to read meaning into numbers—to assess risks, to make and follow budgets, and to understand how planning projections depend on assumptions. As literacy empowered peasants of earlier centuries to exercise personal choice and gain some measure of autonomy, so in our age numeracy offers average citizens the opportunity to make intelligent choices and exercise some degree of control over what otherwise would appear to be a totally mysterious domain of numbers.